Wet dark fluid Cosmological Model in Lyra's Manifold

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Abstract : In this paper, we have obtained field equations and their solution in the presence of wet dark fluid in Lyra's manifold with the aid of Bianchi type-I space time. For solving the Einstein field equations the relation between ρ_{WDF} and p_{WDF} is used. Also, some physical and kinematical properties of the model are discussed.

Key words: Bianchi type-I, Wet dark fluid, Lyra's Manifold.

Introduction:

Lyra (1951) proposed a modification of Riemannian geometry by introducing gauge function into structureless manifold , as a result of which the cosmological constant arises naturally from the geometry. This bears a remarkable resemblance to Weyl's (1918) geometry. In subsequent investigation Sen (1957) and Sen and Dunn (1971) formulated a new scalar-tensor theory og gravitation and constructed an analogue of the Einstein field equations based on Lyra's geometry. Halford (1970,1972) has shown that the scalar-tensor treatment based on Lyra's geometry predicts the same effects as in general relativity.

Bhamra (1974), Kalyanshetti and Waghmode (1982), Reddy and Innaih (1985), Reddy and venkateswarlu (1988) are some of the authors who have investigated various aspects of the four dimensional cosmological models in Lyra's manifold. Singh and Singh (1991,1992,1993) have presented Bianchi Type-I, III and Kantowski-Sachs cosmological models with a time-dependent displacement field and have made a comparative study of the Robertson-Walkar models with a constant deceleration theory based on Lyra's geometry.

Rahaman *et al* (2002), Pradhan and Pandy (2003), stuided some topological defects within the framework of Lyra's geometry, Bhowmik and Rajput (2004) obtained anisotropic Bianchi type cosmological models on the basis of Lyra's geometry. Reddy DRK (2005) examined plane symmetric cosmic strings in Lyra's manifold.

R. Holman and Siddartha Naidu (2005) studied wet dark fluid (WDF) as a model for dark energy. This model was in the spirit of the generalized chaplygin gas (GCG), where a physically motivated equation of state was offered with properties relevant for the dark energy problem. Here the motivation stems from an empirical equation of state proposed by Tait (1988) and Hayword (1967) to treat water and aqueous solution. The equation of state for WDF is very simple.

$$p_{WDF} = \gamma\left(\rho_{WDF} - \rho^{*}\right) \quad (1)$$

and is motivated by the fact that it is a good approximation for many fluids, including water, in which the internal attraction of the molecules makes negative pressures possible. One of the virtues of this model is that the square of the sound speed, C_S^2 which depends on $\frac{\partial p}{\partial \rho}$, can be positive, even while giving rise to cosmic acceleration in the current epoch.

In real fluid negative pressures eventually lead to a breakdown of equation (1) as a Phemenological equation. We will show that this model can be made consistent with the most recent SNIa data, the WMAP results as well as the constraints coming from measurements of the matter power spectrum. The parameters γ and ρ^{*} are taken to be positive and we restrict ourselves to $0 \le \gamma \le 1$. Note that it C_s denotes the adiabatic sound speed in WDF, then $\gamma = C_s^2$.

To find the WDF energy density, we use the energy conservation equation

$$\dot{\rho}_{WDF} + 3H\left(p_{WDF} + \rho_{WDF}\right) = 0 \qquad (2)$$

$$\Rightarrow \qquad \rho_{WDF} = \frac{\gamma}{1+\gamma}\rho^{*} + \frac{D}{V^{(1+\gamma)}} \ ,$$

where D is the constant of integration and V is the volume expansion. WDF naturally includes two components, a piece that behaves as a cosmological constant as well as a piece that red shifts as a standard fluid with an equation of state $p = \gamma\rho$. We can show that it we take $D > 0$, this fluid will never violate the strong energy condition

$p+\rho \geq 0$. Thus, we get

$$p_{WDF}+\rho_{WDF}=(1+\gamma)\rho_{WDF}-\gamma\rho^{*}$$

$$=D(1+\gamma)\frac{D}{V^{(1+\gamma)}} \geq 0 \quad .$$

The wet dark fluid has been used as dark energy in the homogeneous, isotropic FRW case by Holman and Naidu (2005). T. Singh and R. Chaubey (2008) studied in Bianchi type I universe with wet dark fluid. Recently, Adhav *et al* (2011) have been studied in detailed for Einstein-Rosen universe with wet dark fluid.

In this paper, we study the Bianchi type-I model in Lyra's manifold in presence of wet dark fluid .

Metric and solutions of field equations:

We consider the Bianchi type-I metric

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2 \qquad (3)$$

Where A, B ,C are functions of time t only. This ensures that the model is spatially homogeneous.

The relativistic field equations in normal gauge in Lyra's manifold are as

$$R_{ij}-\frac{1}{2}g_{ij}R+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}g_{ij}\phi_k\phi^k=-8\pi T_{ij} \quad (4)$$

Where ϕ_i is a displacement field and the other symbols have their usual meaning as in Riemannian geometry. We now assume the vector displacement field ϕ_i to be the time like constant vector.

$$\phi_i=(0,0,0,\beta=cons\tan t) \qquad (5)$$

The energy momentum tensor for wet dark fluid is given by

$$T_i^j=(\rho_{WDF}+p_{WDF})u_iu^j-p_{WDF}\delta_i^j \quad (6)$$

where u^i is the flow vector satisfying

$$g_{ijj}u^iu^j=1$$

Now, with the help of equation (6), the field equation (4) for the metric (3) can be written as

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}+\frac{3}{4}\beta^2=-8\pi p_{WDF} \qquad (7)$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}+\frac{3}{4}\beta^2=-8\pi p_{WDF} \qquad (8)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}+\frac{3}{4}\beta^2=-8\pi p_{WDF} \qquad (9)$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{3}{4}\beta^2=8\pi\rho_{WDF} \qquad (10)$$

Here the suffix '4' after A,B and C denotes ordinary differentiation with respect to t. Equation (7) to (10) are four

ISSN 2229-5518

independent equations in five unknown $A, B, C, p_{WDF}, \rho_{WDF}$. To get a determinate solution, one extra condition is needed. So we consider the equation of state

$$p_{WDF} = \rho_{WDF} \qquad (11)$$

After solving the set of equation (7) to (10) with the help of equation (11), we obtain we obtain

$$(ABC)_{44} = 0 \qquad (12)$$

Since *A,B,C* are non-zero, Integration of equation (12) gives

$$ABC = (at + b) \qquad (13)$$

Where *a, b* are constants of integration.

Futher Kanser (1921) universe refers to a vacuum cosmological model. The generalizations of Kasner model were proposed by Henekman and schucking (1958).Misner (1968), Lifeshitz-khalatnikov (1970), Belinski (1970,1971), Gron (1985, 1990) has defined an analytic nondimensional expression for the anisotropy of the Kasner metric. Barrow (1997) , Caltaldo (2000) , Brevik and Petterson (1997, 2000) proved that a viscous cosmological fluid does not permit the kasner metric to be anisotropic in Einstein's general relativity, with the help of above the Equation (13) can be written in the following explicit form

$$A = A_0 (at + b)^{p_1}, \quad B = B_0 (at + b)^{p_2}$$

$$C = C_0 (at + b)^{p_3} \qquad (14)$$

With the constant of integration p_1, p_2 and p_3 and A_0, B_0, C_0 satisfy

$$p_1 + p_2 + p_3 = 1 \quad , \quad p_1^{\,2} + p_2^{\,2} + p_3^{\,3} = 1$$

and $A_0 B_0 C_0 = 1$ (15)

Using equation (14), equation (3) becomes

$$ds^2 = dt^2 - A_0^{\,2} (at + b)^{2p_1} dx^2 - B_0^{\,2} (at + b)^{2p_2} dy^2 - C_0^{\,2} (at + b)^{2p_3} dz^2$$

(16)

The metric can be transformed through a proper choice of co-ordinates into the form

$$ds^2 = dT^2 - T^{2p_1} dx^2 - T^{2p_2} dy^2 - T^{2p_3} dz^2 \qquad (17)$$

Which is Bianchi type-I metric of the Kasner form.

Some Physical Properties:

The model (17) represents an exact cosmological model in Lyra's manifold in presence of wet dark fluid. The physical and kinematical parameters of the model (17) are

ISSN 2229-5518

Proper volume $V^3 = \sqrt{-g} = T$

Expansion Scalar $(\theta) = \frac{a}{3T}$

Shear scalar $(\sigma^2) = \frac{7}{162T^2}$

Deceleration parameter $(q) > 0$

Conclusion:

In this paper, we have considered Lyra's field equation in the presence of wet dark fluid for Bianchi type-I space time. For solving the field equations we have used relation between ρ_{WDF} and p_{WDF}. The cosmological model thus obtained represents a radiating universe in Lyra's theory of gravitation.

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